

第2章 1 「接線と法線」 第3回

解答

1. (1) 11 (2) 4 (3) 4 (4) -1
2. (1) $y = 4x + 1$ (2) $y = \frac{3}{2}x + 6$
 (3) $y = x - 2$ (4) $y = x - \frac{\pi}{2}$
3. (1) $\frac{1}{6}$ (2) $\frac{1}{7}$ (3) -2 (4) -1
4. (1) $y = -\frac{1}{3}x + \frac{14}{3}$ (2) $y = -x + 2$
 (3) $y = -4x + 18$ (4) $y = -2x$

解説

1. (1) $f(x) = x^3 - x$ とおくと $f'(x) = 3x^2 - 1$
 $f'(2) = 3 \cdot 2^2 - 1 = 12 - 1 = 11$
 接線の傾きは 11
- (2) $f(x) = \frac{2}{x^2} = 2x^{-2}$ とおくと
 $f'(x) = -4x^{-3} = -\frac{4}{x^3}$
 $f'(-1) = -\frac{4}{(-1)^3} = 4$
 接線の傾きは 4
- (3) $f(x) = 3\sqrt[3]{x^4} = 3x^{\frac{4}{3}}$ とおくと
 $f'(x) = 3 \cdot \frac{4}{3}x^{\frac{1}{3}} = 4\sqrt[3]{x}$
 $f'(1) = 4\sqrt[3]{1} = 4 \cdot 1 = 4$
 接線の傾きは 4
- (4) $f(x) = -e^x$ とおくと $f'(x) = -e^x$
 $f'(0) = -e^0 = -1$
 接線の傾きは -1
2. (1) $f(x) = -x^2 + 2x$ とおくと $f'(x) = -2x + 2$
 $f'(-1) = 2 + 2 = 4$
 接線の方程式は
 $y - (-3) = 4\{x - (-1)\}$ すなわち
 $y = 4(x + 1) - 3 = 4x + 1$
- (2) $f(x) = 6\sqrt{x} = 6x^{\frac{1}{2}}$ とおくと
 $f'(x) = 6 \cdot \frac{1}{2}x^{-\frac{1}{2}} = \frac{3}{\sqrt{x}}$, $f'(4) = \frac{3}{\sqrt{4}} = \frac{3}{2}$
 接線の方程式は $y - 12 = \frac{3}{2}(x - 4)$ すなわち
 $y = \frac{3}{2}(x - 4) + 12 = \frac{3}{2}x + 6$
- (3) $f(x) = x^3 - 2x$ とおくと $f'(x) = 3x^2 - 2$
 $f(1) = 1 - 2 = -1$, $f'(1) = 3 - 2 = 1$
 接線の方程式は $y - (-1) = 1 \cdot (x - 1)$
 すなわち $y = x - 1 - 1 = x - 2$

- (4) $f(x) = -\cos x$ とおくと
 $f'(x) = \sin x$, $f\left(\frac{\pi}{2}\right) = -\cos \frac{\pi}{2} = 0$
 $f'\left(\frac{\pi}{2}\right) = \sin \frac{\pi}{2} = 1$
 接線の方程式は $y - 0 = 1 \cdot \left(x - \frac{\pi}{2}\right) = x - \frac{\pi}{2}$

3. (1) $f(x) = -3x^2 + 3$ とおくと $f'(x) = -6x$
 法線の傾きは $-\frac{1}{f'(1)} = -\frac{1}{-6} = \frac{1}{6}$
- (2) $f(x) = x^4 - x^3$ とおくと $f'(x) = 4x^3 - 3x^2$
 法線の傾きは $-\frac{1}{f'(-1)} = -\frac{1}{-4 - 3} = \frac{1}{7}$
- (3) $f(x) = -\frac{2}{x^2} = -2x^{-2}$ とおくと
 $f'(x) = 4x^{-3} = \frac{4}{x^3}$
 法線の傾きは
 $-\frac{1}{f'(2)} = -1 \div f'(2) = -1 \div \frac{4}{8} = -2$
- (4) $f(x) = 3 \log x$ とおくと $f'(x) = \frac{3}{x}$
 法線の傾きは $-\frac{1}{f'(3)} = -1 \div \frac{3}{3} = -1$

4. (1) $f(x) = \frac{1}{2}x^2 + x$ とおくと $f'(x) = x + 1$
 $f(2) = 2 + 2 = 4$, $f'(2) = 2 + 1 = 3$
 法線の方程式は $y - 4 = -\frac{1}{3}(x - 2)$ すなわち
 $y = -\frac{1}{3}(x - 2) + 4 = -\frac{1}{3}x + \frac{14}{3}$
- (2) $f(x) = -x^3 + 2x^2$ とおくと $f'(x) = -3x^2 + 4x$
 $f(1) = -1 + 2 = 1$, $f'(1) = -3 + 4 = 1$
 法線の方程式は $y - 1 = -\frac{1}{1}(x - 1)$
 すなわち $y = -(x - 1) + 1 = -x + 2$
- (3) $f(x) = \sqrt{x} = x^{\frac{1}{2}}$ とおくと
 $f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$, $f(4) = \sqrt{4} = 2$
 $f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$
 法線の方程式は $y - 2 = \left(-1 \div \frac{1}{4}\right)(x - 4)$
 すなわち $y = -4(x - 4) + 2 = -4x + 18$
- (4) $f(x) = \frac{1}{2} \sin x$ とおくと $f'(x) = \frac{1}{2} \cos x$
 $f(0) = 0$, $f'(0) = \frac{1}{2} \cos 0 = \frac{1}{2}$
 法線の方程式は
 $y - 0 = \left(-1 \div \frac{1}{2}\right)(x - 0) = -2x$
 すなわち $y = -2x$