第１章 3 「導関数の性質」 第３回

解答

1. (1) $y' = 14x$ (2) $y' = 6x^2$

(3) $y' = 2x^3 + \dfrac{1}{3}x$ (4) $y' = 12x + 17$

(5) $y' = 24x^2 + 20x - 2$ (6) $y' = \dfrac{5}{(x+2)^2}$

(7) $y' = -\dfrac{2}{(x-3)^2}$ (8) $y' = \dfrac{2x(x+2)}{(x+1)^2}$

2. (1) $-\dfrac{40}{x^9}$ (2) $-6x^{-3} - 12x^{-5}$

(3) $\dfrac{2}{3}x^{-\frac{5}{6}} - 4x^{-3}$ (4) $\dfrac{3}{4\sqrt[4]{x}}$

(5) $\dfrac{5}{3}\sqrt[3]{x^2}$ (6) $\dfrac{3x+2}{2\sqrt{x}}$

(7) $-\dfrac{x+3}{2\sqrt{x}(x-3)^2}$ (8) $-4(-x+2)^3$

(9) $5(4x+1)^{\frac{1}{4}}$ (10) $\dfrac{3}{2\sqrt[4]{2x-3}}$

(11) $-\dfrac{6}{(3x+2)^3}$ (12) $\dfrac{45}{(-3x+4)^6}$

解説

1. (1) $y' = 7(x^2)' = 7 \cdot 2x = 14x$

(2) $y' = (2x^3)' - (\sqrt{5})' = 6x^2 - 0 = 6x^2$

(3) $y' = \left(\dfrac{1}{2}x^4 + \dfrac{1}{6}x^2\right)' = \left(\dfrac{1}{2}x^4\right)' + \left(\dfrac{1}{6}x^2\right)'$
$= 2x^3 + \dfrac{1}{3}x$

(4) $y' = (3x+1)'(2x+5) + (3x+1)(2x+5)'$
$= 3 \cdot (2x+5) + (3x+1) \cdot 2 = 12x + 17$

(5) $y' = (4x+1)'(2x^2+2x-1)$
$\quad + (4x+1)(2x^2+2x-1)'$
$= 4 \cdot (2x^2+2x-1) + (4x+1) \cdot (4x+2)$
$= 24x^2 + 20x - 2$

(6) $y' = \dfrac{(4x+3)'(x+2) - (4x+3)(x+2)'}{(x+2)^2}$
$= \dfrac{4 \cdot (x+2) - (4x+3) \cdot 1}{(x+2)^2} = \dfrac{5}{(x+2)^2}$

(7) $y' = \dfrac{(2)'(x-3) - 2(x-3)'}{(x-3)^2} = -\dfrac{2}{(x-3)^2}$
または $y' = -2 \cdot \dfrac{(x-3)'}{(x-3)^2} = -\dfrac{2}{(x-3)^2}$

(8) $y' = \dfrac{(2x^2)'(x+1) - (2x^2) \cdot (x+1)'}{(x+1)^2}$
$= \dfrac{4x(x+1) - 2x^2 \cdot 1}{(x+1)^2} = \dfrac{2x^2 + 4x}{(x+1)^2}$
$= \dfrac{2x(x+2)}{(x+1)^2}$

2. (1) $y' = (5x^{-8})' = 5 \cdot (-8x^{-9}) = -40x^{-9}$
$= -\dfrac{40}{x^9}$

(2) $y' = 3 \cdot (-2x^{-3}) + 3 \cdot (-4x^{-5})$
$= -6x^{-3} - 12x^{-5} \left(= -\dfrac{6}{x^3} - \dfrac{12}{x^5}\right)$

(3) $y' = 4 \cdot \left(\dfrac{1}{6}\right) x^{-\frac{5}{6}} + 2 \cdot (-2x^{-3})$
$= \dfrac{2}{3}x^{-\frac{5}{6}} - 4x^{-3} \left(= \dfrac{2}{3\sqrt[6]{x^5}} - \dfrac{4}{x^3}\right)$

(4) $y' = (x^{\frac{3}{4}})' = \dfrac{3}{4}x^{-\frac{1}{4}} = \dfrac{3}{4\sqrt[4]{x}}$

(5) $y' = (x \cdot x^{\frac{2}{3}})' = (x^{\frac{5}{3}})' = \dfrac{5}{3}x^{\frac{2}{3}} = \dfrac{5}{3}\sqrt[3]{x^2}$

(6) $y' = (x+2)'\sqrt{x} + (x+2)(\sqrt{x})'$
$= 1 \cdot \sqrt{x} + (x+2) \cdot (x^{\frac{1}{2}})' = \sqrt{x} + \dfrac{1}{2}(x+2)x^{-\frac{1}{2}}$
$= \sqrt{x} + \dfrac{x+2}{2\sqrt{x}} = \dfrac{2x + (x+2)}{2\sqrt{x}} = \dfrac{3x+2}{2\sqrt{x}}$

(7) $y' = \dfrac{(\sqrt{x})'(x-3) - \sqrt{x}(x-3)'}{(x-3)^2}$
$= \dfrac{\dfrac{1}{2\sqrt{x}}(x-3) - \sqrt{x}}{(x-3)^2} = \dfrac{(x-3) - 2x}{2\sqrt{x}(x-3)^2}$
$= -\dfrac{x+3}{2\sqrt{x}(x-3)^2}$

(8) $y' = -1 \cdot 4(-x+2)^3 = -4(-x+2)^3$

(9) $y' = 4 \cdot \dfrac{5}{4}(4x+1)^{\frac{1}{4}} = 5(4x+1)^{\frac{1}{4}}$
$\left(= 5\sqrt[4]{4x+1}\right)$

(10) $y' = \left\{(2x-3)^{\frac{3}{4}}\right\}' = 2 \cdot \dfrac{3}{4}(2x-3)^{-\frac{1}{4}}$
$= \dfrac{3}{2\sqrt[4]{2x-3}}$

(11) $y' = \{(3x+2)^{-2}\}' = 3 \cdot \{-2(3x+2)^{-3}\}$
$= -\dfrac{6}{(3x+2)^3}$

(12) $y' = \{3(-3x+4)^{-5}\}'$
$= (-3) \cdot \{3 \cdot (-5) \cdot (-3x+4)^{-6}\}$
$= \dfrac{45}{(-3x+4)^6}$