第2章 2 「逆ラプラス変換」 第3回

解答

1. (1) $5t^4e^t$ (2) $t\sin 2t$ (3) $\sinh 3t$ (4) $e^{-2t}\sin t$

2. (1) $-e^t+e^{2t}$ (2) $e^t-\cos 2t+\dfrac{5}{2}\sin 2t$ (3) $-2e^t+3e^{3t}$ (4) $(2-t)e^{-3t}$

(5) $(-5-4t)e^{2t}+5e^{3t}$ (6) $\dfrac{1}{4}+\dfrac{1}{4}(7+18t)\,e^{2t}$

解説

1. (1) $\mathcal{L}^{-1}\left[\dfrac{n!}{(s-\alpha)^{n+1}}\right]=t^ne^{\alpha t}$ より $\mathcal{L}^{-1}\left[\dfrac{5!}{(s-1)^5}\right]=5\mathcal{L}^{-1}\left[\dfrac{4!}{(s-1)^5}\right]=5t^4e^t$

(2) $\mathcal{L}^{-1}\left[\dfrac{2\omega s}{(s^2+\omega^2)^2}\right]=t\sin\omega t$ より $\mathcal{L}^{-1}\left[\dfrac{4s}{(s^2+4)^2}\right]=t\sin 2t$

(3) $\mathcal{L}^{-1}\left[\dfrac{\omega}{s^2-\omega^2}\right]=\sinh\omega t$ より $\mathcal{L}^{-1}\left[\dfrac{3}{s^2-9}\right]=\sinh 3t$

(4) $\mathcal{L}^{-1}\left[\dfrac{\beta}{(s-\alpha)^2+\beta^2}\right]=e^{\alpha t}\sin\beta t$ より $\mathcal{L}^{-1}\left[\dfrac{1}{s^2+4s+5}\right]=\mathcal{L}^{-1}\left[\dfrac{1}{(s+2)^2+1}\right]=e^{-2t}\sin t$

2. (1) $\dfrac{1}{(s-1)(s-2)}=\dfrac{-1}{s-1}+\dfrac{1}{s-2}$ より

$$\mathcal{L}^{-1}\left[\frac{1}{(s-1)(s-2)}\right]=\mathcal{L}^{-1}\left[\frac{-1}{s-1}+\frac{1}{s-2}\right]=\mathcal{L}^{-1}\left[\frac{-1}{s-1}\right]+\mathcal{L}^{-1}\left[\frac{1}{s-2}\right]=-e^t+e^{2t}$$

(2) $\dfrac{6s-1}{(s-1)(s^2+4)}=\dfrac{1}{s-1}+\dfrac{-s+5}{s^2+4}=\dfrac{1}{s-1}-\dfrac{s}{s^2+4}+\dfrac{5}{s^2+4}$ より

$$\begin{aligned}\mathcal{L}^{-1}\left[\frac{6s-1}{(s-1)(s^2+4)}\right]&=\mathcal{L}^{-1}\left[\frac{1}{s-1}-\frac{s}{s^2+4}+\frac{5}{s^2+4}\right]\\&=\mathcal{L}^{-1}\left[\frac{1}{s-1}\right]-\mathcal{L}^{-1}\left[\frac{s}{s^2+4}\right]+\frac{5}{2}\mathcal{L}^{-1}\left[\frac{2}{s^2+4}\right]=e^t-\cos 2t+\frac{5}{2}\sin 2t\end{aligned}$$

(3) $\dfrac{s+3}{s^2-4s+3}=\dfrac{s+3}{(s-1)(s-3)}=\dfrac{-2}{s-1}+\dfrac{3}{s-3}$ より

$$\mathcal{L}^{-1}\left[\frac{s+3}{s^2-4s+3}\right]=\mathcal{L}^{-1}\left[\frac{-2}{s-1}+\frac{3}{s-3}\right]=\mathcal{L}^{-1}\left[\frac{-2}{s-1}\right]+\mathcal{L}^{-1}\left[\frac{3}{s-3}\right]=-2e^t+3e^{3t}$$

(4) $\dfrac{2s+5}{s^2+6s+9}=\dfrac{2s+5}{(s+3)^2}=\dfrac{2}{s+3}+\dfrac{-1}{(s+3)^2}$ より

$$\begin{aligned}\mathcal{L}^{-1}\left[\frac{2s+5}{s^2+6s+9}\right]&=\mathcal{L}^{-1}\left[\frac{2}{s+3}+\frac{-1}{(s+3)^2}\right]=\mathcal{L}^{-1}\left[\frac{2}{s+3}\right]+\mathcal{L}^{-1}\left[\frac{-1}{(s+3)^2}\right]=2e^{-3t}-te^{-3t}\\&=(2-t)e^{-3t}\end{aligned}$$

(5) $\dfrac{s+2}{(s-2)^2(s-3)}=\dfrac{-5}{s-2}+\dfrac{-4}{(s-2)^2}+\dfrac{5}{s-3}$ より

$$\begin{aligned}\mathcal{L}^{-1}\left[\frac{s+2}{(s-2)^2(s-3)}\right]&=\mathcal{L}^{-1}\left[\frac{-5}{s-2}+\frac{-4}{(s-1)^2}+\frac{5}{s-3}\right]\\&=-5\mathcal{L}^{-1}\left[\frac{1}{s-2}\right]-4\mathcal{L}^{-1}\left[\frac{1}{(s-2)^2}\right]+5\mathcal{L}^{-1}\left[\frac{1}{s-3}\right]=-5e^{2t}-4te^{2t}+5e^{3t}\\&=(-5-4t)e^{2t}+5e^{3t}\end{aligned}$$

(6) $\dfrac{2s^2+1}{s^3-4s^2+4s}=\dfrac{2s^2+1}{s(s-2)^2}=\dfrac{1}{4}\dfrac{1}{s}+\dfrac{7}{4}\dfrac{1}{s-2}+\dfrac{9}{2}\dfrac{1}{(s-2)^2}$ より

$$\begin{aligned}\mathcal{L}^{-1}\left[\frac{2s^2+1}{s^3-4s^2+4s}\right]&=\mathcal{L}^{-1}\left[\frac{1}{4}\frac{1}{s}+\frac{7}{4}\frac{1}{s-2}+\frac{9}{2}\frac{1}{(s-2)^2}\right]\\&=\frac{1}{4}\mathcal{L}^{-1}\left[\frac{1}{s}\right]+\frac{7}{4}\mathcal{L}^{-1}\left[\frac{1}{s-2}\right]+\frac{9}{2}\mathcal{L}^{-1}\left[\frac{1}{(s-2)^2}\right]=\frac{1}{4}+\frac{7}{4}e^{2t}+\frac{9}{2}te^{2t}\\&=\frac{1}{4}+\frac{1}{4}(7+18t)\,e^{2t}\end{aligned}$$