第1章 2 「曲線」「曲面」 第2回

解答

1. $\boldsymbol{t}=\left(\dfrac{1}{2}\cos t,\ -\dfrac{1}{2}\sin t,\ \dfrac{\sqrt{3}}{2}\right),\quad s=12$

2. $\boldsymbol{t}=\left(\dfrac{2t^2}{2t^2+1},\ \dfrac{2t}{2t^2+1},\ \dfrac{1}{2t^2+1}\right),\quad s=\dfrac{15}{8}+\log 2$

3. (1) $(2\cos u,\ 2\sin u,\ 0)$　(2) 2　(3) $\pm(\cos u,\ \sin u,\ 0)$　(複号同順)　(4) 16π

解説

1. $\boldsymbol{r}'=(2\cos t,\ -2\sin t,\ 2\sqrt{3})$

$$|\boldsymbol{r}'|=\sqrt{(2\cos t)^2+(-2\sin t)^2+(2\sqrt{3})^2}=\sqrt{16}=4$$

$$\boldsymbol{t}=\frac{\boldsymbol{r}'}{|\boldsymbol{r}'|}=\frac{1}{4}\left(2\cos t,\ -2\sin t,\ 2\sqrt{3}\right)=\frac{\boldsymbol{r}'}{|\boldsymbol{r}'|}=\left(\frac{1}{2}\cos t,\ -\frac{1}{2}\sin t,\ \frac{\sqrt{3}}{2}\right)$$

$$s=\int_0^3\left|\frac{d\boldsymbol{r}}{dt}\right|dt=4\int_0^3 dt=12$$

2. $\boldsymbol{r}'=\left(t,\ 1,\ \dfrac{1}{2t}\right)$

$$|\boldsymbol{r}'|=\sqrt{t^2+1^2+\left(\frac{1}{2t}\right)^2}=\sqrt{\left(t+\frac{1}{2t}\right)^2}=t+\frac{1}{2t}=\frac{2t^2+1}{2t}$$

$$\boldsymbol{t}=\frac{\boldsymbol{r}'}{|\boldsymbol{r}'|}=\frac{2t}{2t^2+1}\left(t,\ 1,\ \frac{1}{2t}\right)=\frac{\boldsymbol{r}'}{|\boldsymbol{r}'|}=\left(\frac{2t^2}{2t^2+1},\ \frac{2t}{2t^2+1},\ \frac{1}{2t^2+1}\right)$$

$$s=\int_{\frac{1}{2}}^{2}\left(t+\frac{1}{2t}\right)dt=\left[\frac{1}{2}t^2+\frac{1}{2}\log t\right]_{\frac{1}{2}}^{2}dt=\frac{15}{8}+\log 2$$

3. (1) $\dfrac{\partial\boldsymbol{r}}{\partial u}\times\dfrac{\partial\boldsymbol{r}}{\partial v}=(-\sin u,\ \cos u,\ 0)\times(0,\ 0,\ 2)=(2\cos u,\ 2\sin u,\ 0)$

(2) $\left|\dfrac{\partial\boldsymbol{r}}{\partial u}\times\dfrac{\partial\boldsymbol{r}}{\partial v}\right|=\sqrt{(2\cos u)^2+(2\sin u)^2+0^2}=2$

(3) $\boldsymbol{n}=\pm\dfrac{\dfrac{\partial\boldsymbol{r}}{\partial u}\times\dfrac{\partial\boldsymbol{r}}{\partial v}}{\left|\dfrac{\partial\boldsymbol{r}}{\partial u}\times\dfrac{\partial\boldsymbol{r}}{\partial v}\right|}=\pm(\cos u,\ \sin u,\ 0)$　(複号同順)

(4) $S=\displaystyle\iint_D\left|\frac{\partial\boldsymbol{r}}{\partial u}\times\frac{\partial\boldsymbol{r}}{\partial v}\right|du\,dv=\int_0^{2\pi}\left\{\int_0^4 2\,dv\right\}du=16\pi$