

解答

1. $\mathbf{t} = \left(\frac{1}{\sqrt{10}} \cos t, -\frac{1}{\sqrt{10}} \sin t, \frac{3}{\sqrt{10}} \right), \quad s = 4\sqrt{10}$
2. (1) $(2v \cos u, 2v \sin u, 0)$ (2) $2v$ (3) $(\pm \cos u, \pm \sin u, 0)$ (複号同順) (4) 6π

解説

1. $\mathbf{r}' = (4 \cos t, -4 \sin t, 12)$
 $|\mathbf{r}'| = \sqrt{(4 \cos t)^2 + (-4 \sin t)^2 + 12^2} = \sqrt{160} = 4\sqrt{10}$
 $\mathbf{t} = \frac{\mathbf{r}'}{|\mathbf{r}'|} = \left(\frac{1}{\sqrt{10}} \cos t, -\frac{1}{\sqrt{10}} \sin t, \frac{3}{\sqrt{10}} \right)$
 $s = \int_1^2 \left| \frac{d\mathbf{r}}{dt} \right| dt = 4\sqrt{10} \int_1^2 dt = 4\sqrt{10}$
2. (1) $\frac{\partial \mathbf{r}}{\partial u} \times \frac{\partial \mathbf{r}}{\partial v} = (-\sin u, \cos u, 0) \times (0, 0, 2v) = (2v \cos u, 2v \sin u, 0)$
 (2) $\left| \frac{\partial \mathbf{r}}{\partial u} \times \frac{\partial \mathbf{r}}{\partial v} \right| = 2v$
 (3) $\mathbf{n} = \pm \frac{\frac{\partial \mathbf{r}}{\partial u} \times \frac{\partial \mathbf{r}}{\partial v}}{\left| \frac{\partial \mathbf{r}}{\partial u} \times \frac{\partial \mathbf{r}}{\partial v} \right|} = \pm \frac{1}{2v} (2v \cos u, 2v \sin u, 0) = (\pm \cos u, \pm \sin u, 0)$ (複号同順)
 (4) $S = \iint_D \left| \frac{\partial \mathbf{r}}{\partial u} \times \frac{\partial \mathbf{r}}{\partial v} \right| du dv = \int_0^{2\pi} \left\{ \int_1^2 2v dv \right\} du = 2\pi [v^2]_1^2 = 6\pi$